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MAXIMUM NEUTRON DENSITY IN A REACTOR WITH LIMITED HEAT-UP OF THE COOLANT

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1. Introduction

The needs of nuclear material engineering for intensive sources of nuclear radiation are growing all the time. Reactors are being constructed in which the materials and assemblies of various devices can withstand ever increasing fluxes of thermal and fast neutrons, which makes it possible to study destruction of the fuel assemblies (Ref.1), obtain trans-uranium elements (Ref.2), investigate the properties of short-lived nuclei (Ref.3). Ten years ago a thermal neutron flux of about $10^{14}cm^{-2}sec^{-1}$ was considered to be very high whereas at present there are reactors which produce short-time fluxes as high as $\sim 10^{16}cm^{-2}sec^{-1}$. It may be supposed that the fluxes will grow to still higher values.

However, since each specific method of obtaining intensive nuclear radiations has its limitations these methods will change in quality. One of the new methods is the use of underground nuclear explosions for obtaining powerful neutron impulses (Ref.4).

Studying the maximum potentialities of the used powerful neutron sources is a main theoretical problem. It will be shown below that for a given amount of energy which can be taken by one volume unit of the coolant there is a constant limit to which the neutron density in the core of the reactor can be increased, regardless of the known reactor type.

The paper decels with pulsed reactors, reactors with fuel motion relative to the core, reactors with transfer of heat from fuel to coolant and reactors with opposing coolant streams.

2. Pulsed reactors

The pulsed reactor keeps operating unitl it heats up to the limit imposed by its construction and the materials. Then the reactor has to cool down before it again becomes operative (the heat loss during the operation is usually negligibly small).

Apart from cooling the operation cycle of a pulsed reactor can be divided into three stages: build-up of power to the maximum level operation at this level, collapse of the power. As the reactor heats up during all three stages, the shorter the duration of each stage, the higher the maximum power.

The rate of neutron density n growth (and, consequently, the rate of power growth) is limited by an exponential curve with a period.

$$T_{min} = \frac{1}{V\Sigma(K_{\infty}-1)} \qquad (1)$$

where $(K_{\infty} -1)$ is the maximum possible supercriticality of the reactor (of infinite dimensions), V - velocity of fission-producing neutrons, Σ -absorbtion cross-section of these neutrons. With such neutron density growth when it reaches maximum the integral (in time) neutron density is equal to

$$\int n dt = T_{min} n_{max} \qquad (2)$$

Accordingly, the heat accumulated per volume unit of the reactor will be equal to $Q = T_{min} n_{max} V\Sigma K_{\infty} \frac{E_f}{Y_f}$. Here, E_f - fission energy, Y_f - neutron yield per fission event, $K_{\infty} E_f/Y_f$ - liberation of energy corresponding to the absorbtion of one neutron.

The value of Q cannot exceed the permissible heat liberation per volume unit of the reactor Q_{max}. The equality of Q and Q_{max} is possible only with the zero duration of the operating duty and instanteneous collapse of power. Therefore the neutron density cannot exceed the value given by

$$n_{max} = \frac{Q_{max} Y_f}{T_{min} V\Sigma K_{\infty} E_f} \qquad (3)$$

which means that in accordance with (1)

$$n_{max} = \left(Y_f - \frac{Y_f}{K_{\infty}}\right) \frac{Q_{max}}{E_f} \qquad (4)$$

Let us represent Q_{max} as the number of atoms per volume unit of the reactor (N) multiplied by the maximum energy which can be on the average imparted to each atom ($\mathcal{E}$):

$$Q_{max} = N\mathcal{E} \tag{5}$$

Then

$$n_{max} = \left(\nu_f - \frac{\nu_f}{K_\infty}\right) \frac{\mathcal{E}}{\mathcal{E}_f} N \tag{6}$$

If the construction of the reactor does not provide for the transition of the core to the plasma state, the value of $\mathcal{E}$ must not exceed several tenths of an electron-volt. Assuming that

$$N \sim 0.5 \cdot 10^{23} cm^{-3}, \ \mathcal{E} \sim 0.3 ev, \ E_f \sim 2 \cdot 10^8 ev, \ \nu_f \sim 2.5, \ K_\infty \sim 2,$$

we obtain

$$n_{max} \sim 10^{14} cm^{-3}$$

The neutron flux depends on the energy of the neutrons which cause the bulk of fissions. In the thermal energy region the cross-section of many important reactions with neutrons follows approximately the 1/V law, therefore "n" is in many cases a sufficient characteristic for thermal-neutron reactors. It is possible, by way of illustration, to calculate the flux referred to a room temperature, regardless of the actual average temperature, i.e. to multiply "n"$_{max}$ by $2.2 \cdot 10^5$ cm/sec. The result is

$$\Phi_{max} \sim 2.10^{19} cm^{-2} sec^{-1}$$

In practice it is rather difficult to shut down instantaneously a pulsed reactor with large super criticality i.e. for the time smaller than T_{min}. If the reactor is shut down due to the negative linear thermal effect the reactivity decrease is proportional to the accumulated heat; n_{max} drops four times. Besides, the real supercriticality is always smaller than K_∞ -1.

The neutron density can be increased beyond $10^{14} cm^{-3}$ only in plasma-type reactors, i.e. by heating the core to 10^4 deg. and above. However, the reactors of such types differ in principle from those which have so far been mastered by the reactor engineering.

It may be mentioned that $\mathcal{E}$ /E_f is the ratio of the fissioned

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nuclei to the full number of the nuclei in the core (Q_{max}/E_f is the number of fission events per volume unit). Consequently $\delta/E_f < 1$. The ultimate limit, which is not achieved even in the atom bomb

$$n_{max}=(Y_f-1)\,N \sim 10^{23}\,cm^{-3}$$

3. Reactors with Fuel Motion Relative to the Core

In these reactors the fuel, thoroughly mixed with liquid or gaseous coolant, moves through the core at a high speed (homogeneous reactors). These reactors also include cyclopiles (Ref.5) in which the zone with a high neutron flux moves along the long reactor body. Since it does not matter what is at rest - the geometrical region where fission is going on (core) or the mixture of the fuel with the coolant, this type of reactor actually differs only in the fact that the coolant may be in any aggregation state and, particularly, in a solid state.

Let us assume that a wave of the neutron flux and of the heating-up travels along the reactor with a velocity U. The absorbers which were used to make the reactor subcritical are removed from the cold reactor body at the forward front of the wave. Absorbers may be inserted into the hot core at the rear front of the wave to shut down the chain reaction so as to avoid excessive heating. If the negative thermal effect is sufficiently large there is no need to use rear absorbers.

The homogeneous reactors are not usually provided with front or rear absorbers. Instead, the cross-section of the stream sharply narrowed, as it leaves the core, the neutron leakage increases and the reaction is shut down over a short section.

For the rough description of the process given below the difference between the insertion of an absorber and narrowing of the cross-section is inessential.

It would seem at a first glance that in this type of reactor the neutron density can be increased limitless even with a given heating limit by merely increasing the velocity U of the motion of the boundary which divides the region of the cold subcritical reactor from the supercritical region of the core.

Of course, this involves engineering problems - the tested materials should steadily keep up with the travelling neutron field, so that they were subjected to irradiation during the time while the wave travels round the reactor body (if the core is stationary the mixture of the fuel and coolant must be circulated at a high velocity). Besides to preserve the duration of irradiation at the higher velocities the reactor length must be increased respectively (if the reactor is to cool down while the wave travells round the reactor the capacity of the cooling system must grow with the velocity U). However, these problems can be solved in principle.

It appars, however, that as the velocity U increases the neutron density first grows in proportion to U but then tends to a certain limit. To illustrate this let us compose a simplified unidimensional single-group equation which describes the distribution of the neutron flux ϕ along the core:

$$\mathcal{D}\left(\frac{\partial^2\phi}{\partial x^2} + \varkappa_x^2\phi\right) = \frac{\partial n}{\partial t} = \frac{1}{V}\cdot\frac{\partial\phi}{\partial t} \qquad (7)$$

Here $\varkappa_x^2$ - the difference between the material parameter and the radial geometric parameter V and D - the velosity and diffusion constant for the fission-producing neutrons.

In the case of a travelling wave the time derivative (ϕ is the function x - ut) can be expressed through the coordinate derivative

$$\frac{\partial\phi}{\partial t} = -u\,\frac{\partial\phi}{\partial x} \qquad (8)$$

We obtain the equation

$$\frac{d^2\phi}{dx^2} + \frac{u}{V\mathcal{D}}\,\frac{d\phi}{dx} + \varkappa_x^2\phi = 0 \qquad (9)$$

Outside the core $\varkappa_x^2$ is a high negative value. Inside the core $\varkappa_x^2$ and VD are, generally speaking, variables as the heating affects the neutron balance and the operating spectrum. This fact is very important when n_{max} has to be calculated for some specific reactor. To find the maximum value n_{max} they can be substituted with some average values.

Let us first assume that in the reactor portions blackened by the absorbers the flux is zero, i.e. $\varkappa_x^2 = -\infty$. Then the

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critical length of the core (the distance between the front and rear absorbers) is equal to

$$a_x = \frac{\pi}{\varkappa_x\sqrt{1-\gamma^2}} \qquad (10)$$

where

$$\gamma = \frac{u}{2\nu D\varkappa_x} \qquad (11)$$

At $\gamma = 1$, obtain $a_x = \infty$. The corresponding velocity U is yet small as compared with the neutron velocity as $D\varkappa_x \ll 1$. At $\gamma > 1$ Eq.(9) (9) has no solution that would become zero at two values of X. This case will be discussed separately.

The distribution of the flux and the neutron density within the reactor is described by the formula

$$n \sim \phi \sim e^{-x\varkappa_x\gamma}\sin\left(x\varkappa_x\sqrt{1-\gamma^2}\right) \qquad (12)$$

(X = 0 - rear boundary, X = a_x - front boundary of the reaction zone.)

At small values of γ the curve is symmetrical, but when γ is close to 1 the maximum is displaced towards the rear front of the wave. The non-uniformity factor of the neutron flux distribution within the core is equal to

$$K_x = \frac{n_{max}}{n_{av}} = \frac{\pi}{\sqrt{1-\gamma^2}}\,\frac{e^{-\frac{\gamma}{\sqrt{1-\gamma^2}}\operatorname{arctg}\frac{\sqrt{1-\gamma^2}}{\gamma}}}{1+e^{-\frac{\gamma}{\sqrt{1-\gamma^2}}\pi}} \qquad (13)$$

The integral neutron density $\int n\,dt$ for some point of the reactor can be represented in the form

$$\int n\,dt = n_{av.}\frac{a_x}{u}$$

where a_x/u - the time taken by the neutron wave to pass the point. Substituting $n_{av.}$ with n_{max} we obtain

$$\int ndt = n_{max}T_{eff} \qquad (14)$$

where

$$T_{eff} = \frac{a_x}{uK_x} \qquad (15)$$

The formula (14) is similar to (2). Therefore, the equation for n_{max} can be obtained by substituting T_{min} in Eq.(3) with T_{eff} - the effective width of the curve n (t):

(16)

Substituting $T_{eff.}$ in Eq.(16) with the corresponding expression given by Eq.(15), substituting U in accordance with Eq.(11) and using Eq.(10) and (13) we obtain

$$n_{max} = \left(V_f - \frac{V_f}{K_\infty}\right)\frac{Q_{max}}{E_f}\,\Psi(\gamma)\,\frac{\mathcal{X}_x^2\,\mathcal{D}}{\Sigma(K_\infty - 1)} \tag{17}$$

where

$$\Psi(\gamma) = \gamma\,\frac{2}{1+e^{-\frac{\gamma}{\sqrt{1-\gamma^2}}\pi}}\,\exp\left(-\frac{\gamma}{\sqrt{1-\gamma^2}}\,\operatorname{arcctg}\frac{\sqrt{1-\gamma^2}}{\gamma}\right) \tag{18}$$

Function Ψ monotonically increases with γ to equal $\frac{2}{e}$ at γ =1. Factor $\frac{\mathcal{X}_x^2\mathcal{D}}{\Sigma(K_\infty - 1)}$ is below unity and becomes unity only if $\mathcal{X}_x^2$ coincides with the material parameter (very great transverse dimensions) and the migration area is fully described by D/Σ .

Let us now take up the case when $\gamma >$ 1. We will write the solution in the form

$$n \sim e^{-x\mathcal{X}_x\gamma}\, sh\left[(x+\delta)\,\mathcal{X}_x\sqrt{\gamma^2-1}\right] \tag{19}$$

It does not become zero at X=0 or X=a_x. However, if δ is sufficiently small and a_x sufficiently great, n will be small at both boundaries of the core. This is in accordance with the actual condition that in the blackened areas $n \neq 0$. The neutron density here depends on power of the available neutron sources (spontaneous fission, external irradiation, at the rear front-delayed neutrons) and on the degree of subcriticality. Naturally, it is very small compared with n_{max}, especially at the forward front. Asymptotically Function (19) has an exponential form

$$\exp\left[-x\,\mathcal{X}_x\left(\gamma - \sqrt{\gamma^2 - 1}\right)\right]$$

Therefore, if at X = a_x the flux is to be by several orders smaller than ϕ_{max} the value $a_x\,\mathcal{X}_x\left(\gamma - \sqrt{\gamma^2-1}\right)$ should be equal to at least a two-digit number.

The non-uniformity factor is now described by the formula

(20)

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For n_{max} we obtain the same formula (17) in which, however

$$\Psi(\gamma) = 2\gamma \exp\left(-\frac{\gamma}{2\sqrt{\gamma^2-1}} \ln \frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}\right), \quad \gamma \geqslant 1 \tag{21}$$

At $\gamma = 1$, Ψ remains equal to $\frac{2}{e}$. As γ rises the function Ψ omnotonically increases tending to unity at $\gamma \to \infty$.

Therefore, the maximum neutron density ($U \to \infty$, $\frac{\varkappa_x^2 D}{\Sigma} \to K_\infty - 1$) does not differ from the maximum neutron density for pulsed reactors. Of course, instantaneous shut down of the reaction by the rear absorbers ($\delta = 0$) is an idealization, as is the instantaneous shut down of a pulsed reactor.

As is the case with pulsed reactor n_{max} decreases if negative thermal effect (decrease of $\varkappa_x^2$ during heat-up) is used instead of forced shut down.

4. Reactors with Heat Transfer from Fuel to Coolant

In reactors of such type high neutron density can be obtained only with small transverse dimensions of the fuel elements, minimum volume of the structural elements and small transverse dimensions of channels with the coolant. Therefore, physically this reactor is almost homogeneous type. Let us represent this reactor schematically in the form of a mixture of nuclei of two kinds. Fuel nuclei with atomic weight A_1 are as a rule at rest. The macroscopic scattering cross-section in the fuel elements is equal to Σ_{s1} and their volume part in the reactor is S_1. The average cosine of the scattering angle in the centre-of-mass system is μ_1. For the coolant whose nuclei move at an average velocity of U_2 the corresponding characteristics are designated by index 2. Then the values

$$P_1 = \frac{S_1 \Sigma_{s1}}{S_1\Sigma_{s1} + S_2\Sigma_{s2}}, \qquad P_2 = 1 - P_1 = \frac{S_2\Sigma_{s2}}{S_1\Sigma_{s1} + S_2\Sigma_{s2}} \tag{22}$$

will be the probabilities of scattering on the fuel and coolant nuclei.

The velocity V_0 of a newly formed neutron may be directed to any side with equal probability. Therefore the average value of the vector V_0 is equal to zero. Assuming that P_1, μ_1 are independent from the neutron energy it is quite easy to calculate the average value of the vector v_k - the velocity of the neutron after

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K collision events (See the Appendix):

$$\bar{V}_{K\,av} = \frac{1-\left[\frac{P_1(1+\mu_1 A_1)}{1+A_1}+\frac{P_2(1+\mu_2 A_2)}{1+A_2}\right]}{1-\left[\frac{P_1(1+\mu_1 A_1)}{1+A_1}+\frac{P_2(1+\mu_2 A_2)}{1+A_2}\right]} \cdot \frac{(1-\mu_2)A_2}{1+A_2} \cdot P_2 \bar{U}_2 \qquad (23)$$

The time taken by the several initial collisions of the neutron is usually short as compared to the neutron life. Therefore, practically during its entire life the neutron in general moves in the direction of the coolant flow (i.e. in the negative direction) at a velocity the absolute value of which es equal to

$$U_{av.} = \frac{1-\frac{1+\mu_2 A_2}{1+A_2}}{1-\frac{P_1(1+\mu_1 A_1)}{1+A_1}-\frac{P_2(1+\mu_2 A_2)}{1+A_2}} P_2 U_2 \qquad (24)$$

As U_2 is small as compared to V, the variation of the absolute velocity of a neutron with time practically is similar to that with slowing down in a stationary medium. Consequently motion of a part of the medium can be well evaluated by evaluating the averaged motion of the neutrons at a velocity U_{av}. An additional term will appear in the balance equation of a stationary reactor, which will be written in the right-hand part:

$$\mathcal{D}\left(\frac{d^2\phi}{dx^2}+\varkappa_x^2\phi\right) = -U_{av.}\frac{dn}{dx} = -\frac{U_{av}}{v}\cdot\frac{d\phi}{dx} \qquad (25)$$

(a similar equation is treated in Ref.6).

Equation (25) is obtained from (9) by substituting U with U_{av}. Therefore it is desirable to redetermine the parameter γ :

$$\gamma = \frac{U_{av.}}{2v\mathcal{D}\varkappa_x} \qquad (26)$$

However, the formula (15) for T_{eff} still contains U_2. As a result an additional factor-ratio U_2/U_{av} appears in the formula (17). Simultaneously, Q_{max} has to be substituted with S_2Q_{max}, i.e. heat accumulated by the coolant has to be distributed throughout the entire core. We obtain

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$$n_{max} = \left(\gamma_f - \frac{\nu_f}{k_\infty}\right) \frac{Q_{max}}{E_f} \cdot \gamma'(\gamma) \frac{\varkappa_x^2 D}{\Sigma(k_{c5} - 1)} \frac{S_2 U_2}{U_{av}} \tag{27}$$

(Σ - the absorbtion cross-section in the core and not in the coolant). To enable the reaction to be maintained at $\gamma > 1$ the coolant entering the reactor must carry at least small quantities of neutrons. Otherwise, even with γ -1 the reaction can not be maintained in the reactor with a finite height, of the core.

To sum up, the maximum neutron density in the reactor with stationary fuel and moving coolant differs from the maximum density in the homogeneous or pulsed reactor by an additional factor S_2U_2/U_{av}. In accordance with (22) and (24) this factor is equal to

$$\frac{S_2 U_2}{U_{av}} = \left[1 + S_1\left(\frac{\Sigma_{s1}}{\Sigma_{s2}} - 1\right)\right] \frac{1 - \frac{P_1(1+\mu_1 A_1)}{1+A_1} - \frac{P_2(1+\mu_2 A_2)}{1+A_2}}{1 - \frac{1+\mu_2 A_2}{1+A_2}} \tag{28}$$

If $S_1 0$ the factor is equal to unity, as would be expected (homogeneous reactor). It is also equal to unity when A_1, μ_1, Σ_{s1} are equal to A_2, μ_2, Σ_{s2}, respectively. In the general case it may differ somewhat from unity, but it is very doubtful that any substantial gain can be obtained from the stationary condition of the fuel (it is not worthwhile to increase $\frac{\Sigma_{s1}}{\Sigma_{s2}}$ at the expense of low density of the coolant since Q_{max} also decreases).

5. Reactors with Opposing Streams

The slower rate of growth of n when the coolant velocity approaches $2VD\varkappa_x$ is due to the increase in the length of the region where the chain reacton is going on; since with ($U \to \infty$) the lengh of the fission reaction zone rises in proportion to U the heat liberation per volume unit of the core no longer grows, i.e. the neutron density n reaches its maximum limit determined by the formula (4). To overcome this limit the amount of neutrons carried away from the reactor by the coolant must be decreased. As was kindly advised to the author by N.N.Ponomarev-Stepnoy this can be achieved by splitting the coolant flow into streams and directing the adjacent streams so that they face each other. Let us find the required degree of the coolant flow splitting. The reactor to be assumed homogeneous. The streams will have rectangular section with sides q_y and q_z times

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smaller than the corresponding dimensions of the reactor a_y and a_z. If the velocities of the streams $\pm U_0$, then, using the Fourier series for a "chessboard" we obtain:

$$u(y,z) = u_0 \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \frac{(4/\pi)^2}{(2s+1)(2t+1)} \sin\left[(2s+1) q_y \frac{\pi}{a_y} y\right] \sin\left[(2t+1) q_z \frac{\pi}{a_z} z\right] \tag{29}$$

This formula must be substituted into the equation

$$\nabla^2\phi + \varkappa^2\phi + \frac{u}{vD}\frac{\partial\phi}{\partial x} = 0 \tag{30}$$

which is true if a_y/q_y and a_z/q_z are great as compared to D.

Let us seek the solution of (30) in the form of a series by the inherent functions of "an unperturbed" operator

$$H_0 = \nabla^2 + \varkappa^2 \tag{31}$$

having the form

$$\psi_{n,l,k}(x,y,z) = \sin\left(n\frac{\pi}{a_x}x\right)\sin\left(l\frac{\pi}{a_y}y\right)\sin\left(k\frac{\pi}{a_z}z\right) \tag{32}$$

The corresponding inherent values are equal to

$$\lambda_{n,l,k} = \delta\varkappa^2 - (n^2-1)\frac{\pi^2}{a_x^2} - (l^2-1)\frac{\pi^2}{a_y^2} - (k^2-1)\frac{\pi^2}{a_z^2} \tag{33}$$

where

$$\delta\varkappa^2 = \varkappa^2 - \left(\frac{\pi^2}{a_x^2} + \frac{\pi^2}{a_y^2} + \frac{\pi^2}{a_z^2}\right) \tag{34}$$

is the variation of the critical material parameter caused by "perturbation" – motion of the coolant (the perturbation operator is equal to $\delta H = \frac{u}{vD}\cdot\frac{\partial}{\partial x}$).).

For expansion factors ϕ by $\psi_{n,l,k}$ we obtain, in the

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usual way a set of equations in the form:

$$\lambda_{n,l,k}\, C_{n,l,k} + \sum_{m,p,u} \delta H_{n,l,k;\,m,p,u}\, C_{m,p,u} = 0 \qquad (35)$$

The matrix element contained in this equation is determined by the relation

$$\delta H_{n,l,k;\,m,p,u} = \int \psi_{n,l,k} \frac{u}{\nu D} \frac{\partial}{\partial x} \psi_{m,p,u}\, dx\,dy\,dz \Big/ \int \psi^2_{n,l,k}\, dx\,dy\,dz$$

and is equal to

$$\delta H_{n,l,k;m,p,u} = \frac{\pi}{a_x} \frac{u_0}{\nu D} \left(\frac{4}{\pi}\right)^3 \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \Delta(m+n)\, \Delta(l+p+q_y)\, \Delta(k+u+q_z)\,\cdot$$

$$\cdot \frac{mn}{n^2-m^2} \frac{l\,p\,k\,u\, q_y q_z}{\left[(2s+1)^2 q_y^2-(l+p)^2\right]\left[(2s+1)^2 q_y^2-(l-p)^2\right]\left[(2t+1)^2 q_z^2-(k+u)^2\right]\left[(2t+1)^2 q_z^2-(k-u)^2\right]} \qquad (36)$$

where

$$\Delta(h) = \begin{cases} 1 & \text{if } h \text{ is an odd number} \\ 0 & \text{if } h \text{ is an even number} \end{cases} \qquad (37)$$

The equations become simpler at great q_y, q_z, when approximately

$$C_{n,l,k} = \frac{\frac{\pi}{a_x} \frac{u_0}{\nu D}}{(n^2-1)\frac{\pi^2}{a_x^2} + (l^2-1)\frac{\pi^2}{a_y^2} + (k^2-1)\frac{\pi^2}{a_z^2} - \delta\varkappa^2} \sum_{m,p,u} \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \left(\frac{4}{\pi}\right)^3 \Delta(m+n) \frac{mn}{n^2-m^2} \qquad (38)$$

$$\cdot\, \Delta(p+l+q_y)\, \Delta(k+u+q_z) \frac{C_{m,p,u}}{8^2(2s+1)(2t+1)}\,\cdot$$

$$\cdot \left[\frac{1}{(2s+1)q_y-(p+l)} - \frac{1}{(2s+1)q_y-(p-l)}\right]\left[\frac{1}{(2t+1)q_z-(k+u)} - \frac{1}{(2t+1)q_z-(k-u)}\right]$$

It can be easily seen that if values n, $|(2s+1)q_y-1|$, $|(2t+1)q_z-k|$ are of the order of the unity, then

$$C_{n,l,k} \sim \frac{C_{1,1,1}}{(2s+1)(2t+1)} \frac{\frac{\pi}{a_x} \frac{u_0}{\nu D}}{(2s+1)^2 q_y^2 \frac{\pi^2}{a_y^2} + (2t+1)^2 q_z^2 \frac{\pi^2}{a_z^2}}$$

If n,l,k are of the order of the unity, then

$$C_{n,l,k} \sim C_{1,1,1} \frac{\frac{\pi}{a_x} \frac{u_0}{\nu D}}{q_y^2 \frac{\pi^2}{a_y^2} + q_z^2 \frac{\pi^2}{a_z^2}}$$

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If n=l=k=1, only those terms are essential in the right-hand part of (38) in which m_1 $|(2s+1)q_y-P|$, $|(2t+1)q_z-U|$ is of the order of the unity.

Bearing in mind these considerations the following formula can be easily obtained to change $\varkappa^2$ compensating for the entrainment and carry-over of the neutrons by the flow:

$$\frac{\delta\varkappa^2}{(\pi/a_x)^2}=\frac{\gamma_{str}}{16}\left[\sum_{s=0}^{\infty}\sum_{t=0}^{\infty}\frac{q_y^2\frac{\pi^2}{a_y^2}+q_z^2\frac{\pi^2}{a_z^2}}{(2s+1)^2q_y^2\frac{\pi^2}{a_y^2}+(2t+1)^2q_z^2\frac{\pi^2}{a_z^2}}\cdot\frac{1}{(2s+1)^2(2t+1)^2}\right] \tag{39}$$

where

$$\gamma_{str}=\frac{u_0}{2\sqrt{D}\,\varkappa_{str}} \tag{40}$$

and

$$\varkappa^2_{str}=q_y^2\frac{\pi^2}{a_y^2}+q_z^2\frac{\pi^2}{a_z^2} \tag{41}$$

is the geometric parameter of one stream.

The factor in square brackets in (39) does not practically differ from unity so that the effect of the neutron entrainment on the critical size is insignificant if $\gamma_{str} < 1$.

Disregarding the factor which is close to unity and considering (34) we write (39) as follows:

$$\frac{(\pi/a_x)^2}{\varkappa^2}=\frac{1-\varkappa_z^2/\varkappa^2}{1+\frac{(\gamma_0/4q)^2}{\varkappa_z^2/\varkappa^2}}, \tag{42}$$

where

$$\varkappa_z^2=\frac{\pi^2}{a_y^2}+\frac{\pi^2}{a_z^2} \tag{43}$$

$$\gamma_0=\frac{u_0}{2\sqrt{D}\,\varkappa} \tag{44}$$

$$q^2=\frac{q_y^2\frac{\pi^2}{a_y^2}+q_z^2\frac{\pi^2}{a_z^2}}{\frac{\pi^2}{a_y^2}+\frac{\pi^2}{a_z^2}}=\frac{\varkappa^2_{str}}{\varkappa_z^2} \tag{45}$$

The value q characterizes the degree of splitting the flow into streams. For each value of the parameter ($\gamma_0/4q$) there is

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an optimum value of $\mathcal{X}^2_\gamma/\varkappa^2$ at which a_x is closest to the minimum value of $\mathcal{N}/X$, i.e. the neutron density is maximum. To ensure that the difference between a_x and $\mathcal{N}/\varkappa$ is not great the parameter (γ_o /4q) must not exceed unity. Whence the required degree of splitting[x]:

$$q \sim \gamma_o/4 \qquad (46)$$

For example, to raise the neutron density by one order higher as to its maximum value in a pulsed reactor γ_o must be approximately 15, Q q, approximately 4.

6. Conclusion

It is not an easy problem to obtain neutron densities close to the maximum values determined by the formulas (4) and (27). With fast-neutron reactors this becomes downright impossible since there are no methods by which the reaction can be shut down during the time T_{min} determined by the formula (1). Even scattering of the materials due to their thermal expansion or evaporation does not proceed quickly enough (naturally, at $\mathcal{E} <$ 1 ev). The theoretical limit can be reached more easily in a pulsed resonance reactor.

With thermal-neutron reactors the theoretical limit can be closely approached not only in a pulsed reactor but also in a homogeneous reactor. The required velocities of relative motion of the core and the reactor body come close to 10^2 m/sec (at $\gamma \sim 1$). For reactors in which the fuel is separated from the coolant such velocities are less feasible than for homogeneous reactors as the fuel elements must withstand the coolant flow but their thickness must be reduced to the minimum to avoid excessive internal overheating Further increase of the velocity ($U > 2VD\varkappa$) will be of no avail unless opposing streams of the coolant are used. In a cyclopile this complication of design is very important. So, inside the reactor with a non-ionized coolant the neutron density of order $10^{14}cm^{-3}$ can be obtained only in case the neutron spectrum is thermal.

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*Strictly speaking with $\gamma_o/4q \sim 1$ formula (39) cannot be applied.

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However in the reflector a higher neutron density than $10^{14} cm^{-3}$ is avilable by forcing the resonant reactor.

With resonant spectrum $\gamma = 1$, when U is measured by kilometers in second. Therefore resonant neutron carryover is not important, the length of the core of a resonant reactor is independent on U and thermal stress is limited only by a permissible velocity of the fuel and the core motion. The neutron density in the reflector, static to the core, does not directly depend on fuel concentration in the core, i.e. hardness of spectrum. This density may be 1-2 orders more the neutron density in the core of the resonant reactor and the limit established by formula (4) is not valid for it.

But one cannot count on a high gain compared to the thermal reactor. In the reflector heat is liberated which is to be removed.

So thermal neutron carryover by a coolant is also limiting the available flux.

For many purposes, merely obtaining high neutron density is not enough: this density must be retained for a comparatively long period of time. Obviously, in this respect the cyclopile or homogeneous reactor are superior to the pulsed reactors. It also quite clear that production of a high neutron flux to be maintained for long periods of time with minimum expenditures places specific requirements on the selection of the core materials, its size and construction.

7. Appendix

Assume that V_c and V_c' are the neutron velocities in the centre-of-mass system before and after collision with a nucleus whose velocity and mass number in a laboratory coordinate system are u and A, respectively. As $V_c = V_c'$ it can be assumed that V_c' results from the effect produced on V_c by some random turn operator i.e.

$$\bar{V}_c' = \theta \bar{V}_c \qquad (1)$$

The average cosine of the turn angle is μ, i.e.

$$(\bar{V}_c \theta \bar{V}_c)_{av.} = \mu \bar{V}_c^2 \qquad (2)$$

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As the average value of the vector $\theta \bar{V}_c$ is the vector parallel to V_c, from (2) it follows

$$(\theta \bar{V}_c)_{av.} = \mu \bar{V}_c \tag{3}$$

Take notice, θ is a linear operator.
The neutron velocities in the laboratory system before and after collision ($\bar{V}$ and $\bar{V}'$) are related as follows

$$\bar{V}' = \frac{\bar{V} + A\bar{u}}{1+A} + \theta \frac{A(\bar{V}-\bar{u})}{1+A} = \frac{1+A\theta}{1+A}\bar{V} + \frac{A(1-\theta)}{1+A}\bar{u} \tag{4}$$

If there are two kinds of nuclei and the probabilities of collision with them are equal to P_1 and P_2 then

$$\bar{V}' = L\bar{V} + \bar{O} \tag{5}$$

where L and O with probabilities P_i are respectively equal to

$$\mathcal{L}_i = \frac{1 + A_i\theta_i}{1 + A_i} \tag{6}$$

$$\bar{O}_i = \frac{A_i(1-\theta_i)}{1+A_i}\bar{u}_i \tag{7}$$

From the formula (3) it follows that by averaging (5) with respect to collision the operator θ_i can be substituted with μ_i so that

$$\bar{V}'_{av} = L_{av}\bar{V} + \bar{O}_{av} \tag{8}$$

where

$$L_{av} = P_1L_{1av} + P_2L_{2av}, \quad \bar{O}_{av} = P_1\bar{O}_{1av} + P_2\bar{O}_{2av}$$

and $L_{i\ av}$ and $O_{i\ av}$ are obtained from (6) and (7) by substituting θ_i with μ_i, i.e.

$$L_{av} = P_1\frac{1+A_1\mu_1}{1+A_1} + P_2\frac{1+A_2\mu_2}{1+A_2} = 1-\left[P_1\frac{A_1(1-\mu_1)}{1+A_1} + P_2\frac{A_2(1-\mu_2)}{1+A_2}\right] \tag{9}$$

$$\bar{O}_{av} = P_1\frac{A_1(1-\mu_1)}{1+A_1}\bar{u}_1 + P_2\frac{A_2(1-\mu_2)}{1+A_2}\bar{u}_2 \tag{10}$$

Now, let us take a neutron with a randomly oriented velocity $\bar{V}_o$ after K collisions. Its velocity V_k can be obtained by applying K

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times the formula (5). The average value $V_{k\ av}$ is obtained by subsequent averaging for all collisions and averaging for initial direction V_0. However, since the collisions are statistically independent (we take that P_i and μ_i are independent on energy) the same result can be reached by applying K times the formula (8), assuming that $\bar{V}_{0\ av} = 0$. We obtain:

$$\bar{V}_{c\ av} = \bar{O}_{av.} + \mathcal{L}_{av.}\left\{\bar{O}_{av.} + \mathcal{L}_{av.}\left[\bar{O}_{av} + \cdots + \mathcal{L}_{av.}\left(\bar{O}_{av} + \mathcal{L}_{av}\bar{V}_{0\ av}\right)\ldots\right]\right\} = \tag{11}$$

$$= \bar{O}_{av.} + \mathcal{L}_{av}\bar{O}_{av} + \mathcal{L}^2_{av.}\bar{O}_{av.} + \cdots + \mathcal{L}^{K-1}_{av.}\bar{O}_{av.} = \frac{1-\mathcal{L}^{K}_{av.}}{1-\mathcal{L}_{av}}\bar{O}_{av.}$$

or:

$$\bar{U}_{av} = \left[1 - \left(P_1\frac{1+\mu_1 A_1}{1+A_1} + P_2\frac{1+\mu_2 A_2}{1+A_2}\right)^K\right]\frac{P_1\frac{A_1}{1+A_1}(1-\mu_1)\bar{U}_1 + P_2\frac{A_2}{1+A_2}(1-\mu_2)\bar{U}_2}{P_1\frac{A_1}{1+A_1}(1-\mu_1) + P_2\frac{A_2}{1+A_2}(1-\mu_2)}$$

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